OP AMP PERFORMANCE ANALYSIS

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Given the numerous specifications describing op amp performance, the above title suggests an ambitious goal for one bulletin. Yet, this bulletin reflects the analysis power gained through knowledge of an op amp circuit's feedback factor. Feedback dictates the performance of an op amp both in function and in quality. The major specifications of the amplifier describe an open-loop device awaiting feedback direction of the end circuit's function. Just how well the amplifier performs the function reflects through the feedback interaction with the open-loop error specifications. Fortunately, most open-loop errors simply reflect to the circuit output amplified by the reciprocal of the circuit's feedback factor.

Amplifier bandwidth limits this simple relationship but the feedback factor defines this limit as well. Above a certain frequency, the amplifier lacks sufficient gain to continue amplification of signal and errors alike. Graphical analysis defines this frequency limit through plots representing available amplifier gain and the feedback demand for that gain. This same analysis indicates frequency stability characteristics for op amp circuits. Just the slopes of the plots indicate the phase shift in the feedback loop. Thus, the feedback factor of an op amp circuit is a powerful performance indicator.

The determination of a circuit's feedback factor depends upon feedback modelling. The basic feedback model of an op amp applies directly to the noninverting circuit configuration. Using this configuration, this treatment demonstrates the performance, feedback and stability concepts common to all op amp configurations. A simple guideline extends feedback factor determination to most other op amp circuits. Just knowing a circuit's feedback factor extends the concepts and conclusions of this bulletin to these other op amp configurations.

FEEDBACK FACTOR DEFINES PERFORMANCE

More than any other parameter, the feedback factor of an op amp application defines the circuit performance.[1] Feedback factor sets the gain received by the input-referred errors of the amplifier. These open-loop errors include offset voltage, noise and the error signals generated by limitations in open-loop gain, common-mode rejection and power-supply rejection. In addition, a circuit's feedback factor determines bandwidth and frequency stability.

For the noninverting op amp configuration, a convenient relationship between closed-loop gain and feedback factor simplifies performance analysis. There, the gain of the application circuit itself sets the amplification of input-

referred errors and determines the circuit bandwidth. Shown in Figure 1 as a voltage amplifier, this noninverting circuit produces the familiar, ideal closed-loop gain of $A_{CLi} = (R_1 + R_2)/R_1$. This gain amplifies both the input signal e_i and the differential input error e_{id} of the op amp. Simply multiplying e_{id} by A_{CLi} defines the resulting output error. Later examination adds frequency dependence to this simple relationship.
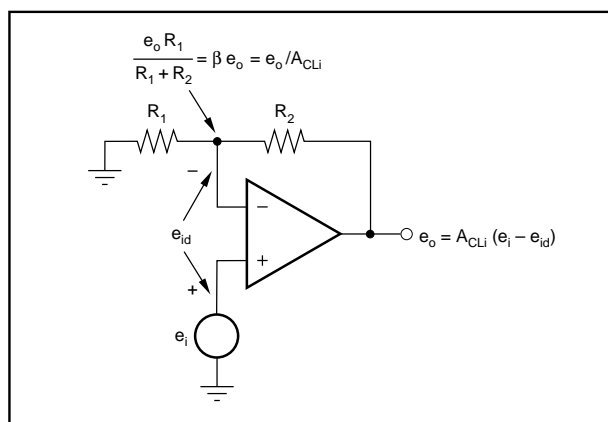

FIGURE 1. Noninverting op amp connections amplify input signal e_i and error signal e_{id} by a gain of $A_{CLi} = 1/\beta$.

The fundamental mechanism relating input and output errors lies in the feedback factor. Feedback factor is the fraction of the amplifier output signal fed back to the amplifier input. In the figure, a feedback voltage divider defines this fraction through the output to input transfer response

$$\beta e_o = \frac{e_o R_1}{R_1 + R_2}$$

This defines β as simply the voltage divider ratio, $R_1/(R_1 + R_2)$. Comparison of this result with A_{CLi} shows that $A_{CLi} = 1/\beta$ for the noninverting case.

Other op amp circuit configurations produce different A_{CLi} but β remains the same. As a general guideline, the feedback factor of an op amp circuit equals the voltage divider ratio of the feedback network. This fact extends the results developed below with the noninverting circuit to almost all other op amp circuits. Just determining this voltage divider ratio for a circuit defines the β term common to a broad range of performance results. In rare cases, complex feedback defies this simple guideline, requiring detailed feedback modelling.[1]

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General error analysis depends on β rather than A_{CLi} as emphasized with the model of Figure 2. This model represents the noninverting op amp connection by an amplifier with input error signal e_{id} and with feedback transmission factor β. This feedback factor determines the signal βe_o fed back to the amplifier input from the output signal e_o. Writing a loop equation for the model shows that

$$e_o = (1/\beta)\,(e_i - e_{id})$$

In this result, a gain of $1/\beta$ amplifies both e_i and e_i. Thus, the Figure 1 circuit and Figure 2 model agree for purposes of input-to-output transmission of amplifier signals.

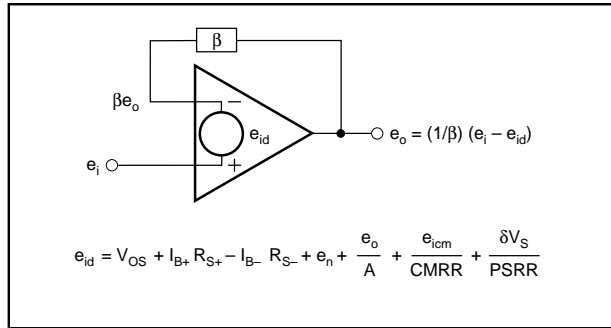

FIGURE 2. The input error amplified by $1/\beta$, e_{id}, includes the effects of the major performance characteristics of an op amp.

INPUT-REFERRED ERRORS SUMMARIZE PERFORMANCE

The simple $1/\beta$ relationship between input and output errors predicts the output errors resulting from almost all amplifier performance characteristics. Each of these characteristics produces an input-referred error source for the op amp as combined in

$$\begin{aligned} e_{id} = {} & V_{OS} + I_{B+}\,R_S + I_{B-}\,R_{S-} + e_n + e_o/A \\ & + e_{icm}/CMRR + \delta V_S/PSRR \end{aligned}$$

Error terms included here cover the effects of the op amp input offset voltage, input bias currents, input noise voltage, open-loop gain, common-mode rejection and power-supply rejection. Here, the second and third terms of the e_{id} equation include the source resistances presented to the two amplifier inputs. The last three error terms include circuit signals which are the output voltage, the common-mode voltage and the power supply voltage change.

The input-referred representations of the individual error terms generally follow from the definitions of the associated performance characteristics. Definitions directly classify V_{OS}, I_{B+}, I_{B-} and e_n as input error sources. Open-loop gain is simply a ratio of output voltage to differential input voltage. Dividing the output voltage by the gain defines the associated input signal as e_o/A. The amplifier's finite open-loop gain requires this input error signal to support the output signal. Similarly, power-supply rejection ratio equals the

ratio of a power supply change to the resulting change in differential input voltage. Thus, $PSRR = \delta V_S/\delta e_{id}$ and the associated input-referred error is $\delta e_{id} = \delta V_S/PSRR$.

For CMRR, the relationship between definition and input error requires closer examination. Common-mode rejection ratio is defined as the ratio of the differential gain to the common-mode gain, A_D/A_{CM}. For an op amp, the differential gain is simply the open-loop gain A. Then, $CMRR = A/A_{CM}$ and rewriting this shows the common-mode gain to be $A_{CM} = A/CMRR$. However, by definition $A_{CM} = e_{ocm}/e_{icm}$ where e_{ocm} is the output signal resulting from e_{icm}. Combining the two A_{CM} equations results in $e_{ocm} = Ae_{icm}/CMRR$. To support this component of output voltage, the op amp develops another gain error signal in e_{id}. As before, the resulting e_{id} component equals the associated output voltage divided by the open-loop gain. Dividing the preceding e_{ocm} expression by open-loop gain A defines the input-referred CMRR error as $e_{icm}/CMRR$.

Closer examination also clarifies the source resistances, R_{S+} and R_{S-}, of the e_{id} equation. In the simplest case, a source resistance is just the output resistance of a signal source that drives a circuit input. For op amp circuits, scaling and feedback resistances alter the net resistances presented to the amplifier's inputs. The difference amplifier connection well illustrates this as shown in Figure 3. There, scaling resistors R_3 and R_4 alter the resistance presented to the amplifier's noninverting input and feedback resistors R_1 and R_2 alter that presented to the inverting input.
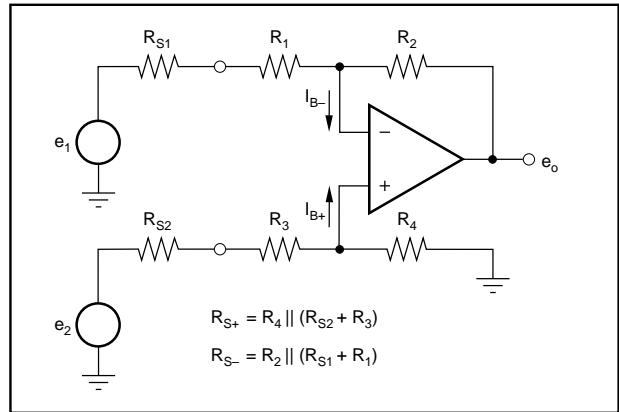

FIGURE 3. Scaling and feedback resistors alter the source resistances presented to the input bias currents of an op amp.

Signal sources e_1 and e_2 drive the difference amplifier's inputs through conventional source resistances R_{S1} and R_{S2}. However, the scaling and feedback resistances alter the net resistances presented to the op amp's input currents I_{B+} and I_{B-}. Current I_{B+} divides between two paths to ground through R_4 and the $R_3 + R_{S2}$ combination. Here, the $R_3 + R_{S2}$ path returns to ground through the low resistance of the e_2 source. Thus, for the e_{id} equation, $R_{S+} = R_4 \| (R_3 + R_{S2})$. Analogously, I_{B-} divides between the path through R_2 and that through $R_1 + R_{S1}$. In this case, R_2 departs from the analogy by returning

to the op amp output instead of to ground. However, the low output impedance of the op amp produces an equivalent result for this resistance evaluation. Thus, $R_{S-} = R_2 \| (R_1 + R_{S1})$.

Together, the error terms of the Figure 2 model provide a fairly complete representation of op amp performance limits. However, the e_{id} expression does not specifically list errors due to distortion, bandwidth and slew rate limiting. Actually, e_{id} includes the amplifier's distortion error in the gain and CMRR error signals.[2] A circuit's bandwidth limit restricts the effects of the e_{id} error sources at higher frequencies. Slew rate limiting simply imposes a secondary bandwidth limit for large signal operation. Feedback factor analysis treats the bandwidth limiting of error effects later.

Up to the circuit's bandwidth limit, each input-referred error term of the Figure 2 model reflects to the amplifier output through a gain equal to $1/\beta$. Multiplication of the error terms by $1/\beta$ produces some familiar results. Output error due to the finite open-loop gain becomes $e_o/A\beta$. This shows that error due to open-loop gain reduces the output e_o by a fraction of that output. This fraction equals the reciprocal of the loop gain $A\beta$. The decline of A with frequency makes this error rise and this shapes the closed-loop frequency response of the circuit. Similar multiplication of the input noise error defines the output noise as e_n/β, leading to the term "noise gain" for $1/\beta$. This description of $1/\beta$ only holds under the bandwidth limits to be described. For both the loop gain and noise errors, greater visibility results through the frequency response analysis described below. Similarly, the frequency dependencies of CMRR and PSRR reflect to the circuit output with circuit-specific bandwidths.

FEEDBACK MODELLING DEFINES CLOSED-LOOP RESPONSE

The above discussion presents the $1/\beta$ relationship between input-referred op amp error sources and the resulting output errors. However, the frequency dependance of amplifier gain modifies this simple, initial relationship. Amplifier response roll off defines a bandwidth limit for both signal and error sources. This reduces the output error effect of all error sources except for the DC errors V_{OS}, $I_{B+} R_{S+}$ and $I_B R_{S-}$. Amplifier gain, noise, CMRR and PSRR produce AC errors and their output effects depend on the circuit's frequency response. More complete feedback modelling defines this frequency response through the noninverting amplifier example. However, the frequency response results developed here extend to any op amp configuration through a standardized response denominator.

Figure 4 shows the generalized noninverting connection with the feedback network as the generalized Z_1 and Z_2 rather than the resistors shown before. Redrawing the amplifier configuration as shown highlights the voltage divider action of the feedback network. The network's divider action again displays the fraction of the amplifier output fed back to the amplifier input. In preparation for the next modelling step, the figure reduces the amplifier input error signal, e_{id}, to just the open-loop gain error e_o/A. Feedback

modelling focuses on gain and related frequency characteristics. Still, this one error signal suffices to define frequency response for use with the previous multi-error analysis.
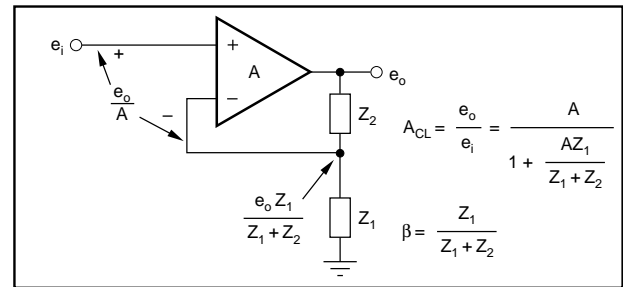

FIGURE 4. Redrawing the op amp circuit and reducing e_{id} to the gain error signal, e_o/A, prepares the circuit for feedback modelling.

Loop analysis defines the noninverting circuit's transfer response as

$$ A_{CL} = \frac{e_o}{e_i} = \frac{A}{1 + \dfrac{AZ_1}{Z_1 + Z_2}} $$

Gain A in this expression contains the frequency dependence that shapes the circuit's frequency response. Note that the denominator of this response contains the feedback factor $Z_1/(Z_1 + Z_2)$. This makes the denominator $1 + A\beta$ and this relates the circuit to the model presented next.

To more completely model the noninverting circuit, Figure 5 replaces the op amp of Figure 4 with a gain block and a summation element. Also, a feedback block replaces the feedback network from before. The gain block represents the amplifier open-loop gain and the summation models the differential action of the op amp inputs. Op amp open-loop gain amplifies the differential signal between the two amplifier inputs. Opposite polarities at the model's summation inputs reproduce the differential action in the summation. Here, the polarity assignments match the polarities of the corresponding op amp inputs. With these assignments, the summation extracts the differential signal through subtraction. The model then supplies the differential signal to the gain block and this block drives the feedback block β. For op amps, this classic feedback model, initially developed by Black[3], only represents the noninverting case. Modifications to the model adapt it to other configurations.[1] However, the noninverting case here suffices to define performance conditions common to all op amp configurations.
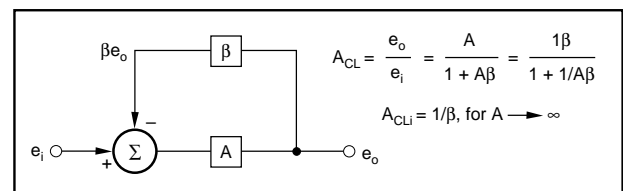

FIGURE 5. Black's classic feedback model reproduces the A_{CL} transfer response of the noninverting op amp configuration.

Comparison of circuit and model responses demonstrates the model validity. The model amplifies the difference between the summation inputs by gain A to produce the output signal. This results in $e_o = A(e_i - \beta e_o)$ and solving for e_o/e_i defines the modelled transfer response as

$$A_{CL} = \frac{e_o}{e_i} = \frac{A}{1 + A\beta}$$

Comparison of terms in the A_{CL} equations for the model, above, and the circuit, before, shows the feedback factor to be $\beta = Z_1/(Z_1 + Z_2)$, validating the model.

LOOP GAIN SUSTAINS RESPONSE

Further analysis of the A_{CL} result defines the op amp frequency response and stability conditions.[4] This added performance information depends upon the denominator of the A_{CL} response and not upon the noninverting case considered here. Conclusions based upon this denominator extend to all other op amp configurations. Rewriting the A_{CL} equation for the noninverting case yields

$$A_{CL} = \frac{1/\beta}{1 + 1/A\beta}$$

Then, the response numerator expresses the ideal closed-loop gain, $A_{CLi} = 1/\beta$, and the denominator expresses the frequency dependance in through A and β.

Other op amp configurations produce different numerators, but always with the same $1 + 1/A\beta$ denominator. This common denominator unifies bandwidth and stability characteristics for all op amp configurations. All op amp configurations produce a closed-loop response of

$$A_{CL} = \frac{A_{CLi}}{1 + 1/A\beta}$$

Writing a given configuration's response in this form immediately identifies the ideal response, A_{CLi}, as the numerator. It also directly links the configuration to the denominator-based bandwidth results and stability criteria that follow.

The frequency dependencies of A and β combine to set a configuration's frequency response. At low frequencies, the high level of open-loop gain A reduces the denominator above to $1 + 1/A\beta \approx 1$. Then, the circuit response simplifies to the ideal gain of A_{CLi}. At higher frequencies, the op amp open-loop gain drops, causing this denominator to increase. Then, A_{CL} declines from it's ideal value A_{CLi}. Similarly, a high-frequency drop in β would add to the A_{CL} decline. Initially, a constant β simplifies the analysis. Constant β results with the resistive feedback networks common to the majority of applications.

The open-loop gain decline with frequency produces the circuit's bandwidth limit as illustrated in Figure 6. There, the resistive feedback case illustrates the most common condition. Reactive rather than resistive feedback slightly modifies the bandwidth conclusions developed here and a later example describes this effect. However, reactive feedback does not alter the frequency stability conditions developed

through this resistive feedback example. The plot of the figure displays the frequency responses of all three variables in the A_{CL} equation. Shown are the closed-loop gain, A_{CL}, the open-loop gain, A_{OL}, and $1/\beta$ as a function of frequency. The graphical interaction of these variables provides visual insight into bandwidth and frequency stability limits. The heavier curve represents the resulting closed-loop response A_{CL}.
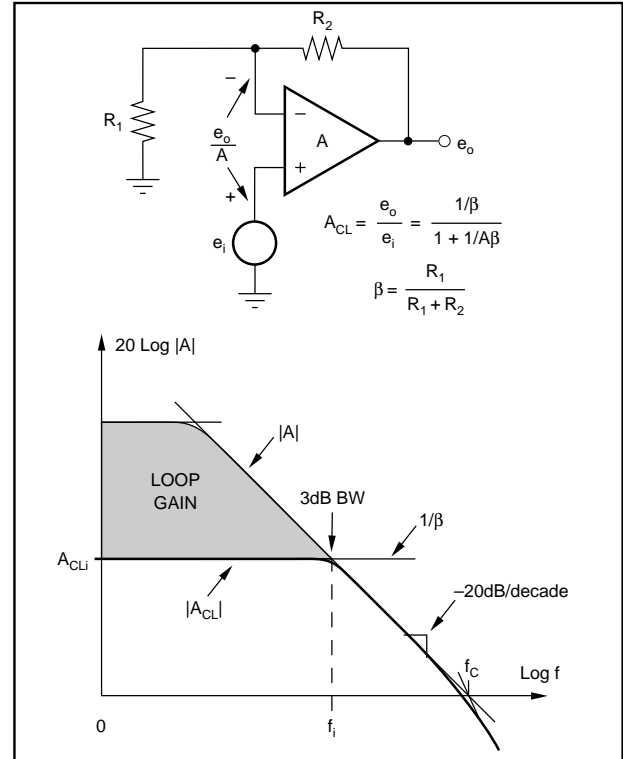

FIGURE 6. Graphical analysis with a circuit's A and $1/\beta$ curves defines the circuit's closed-loop bandwidth.

The loop gain of the circuit, $A\beta$ in the above denominator, represents the amplifier gain resource available to maintain the ideal closed-loop response. In Figure 6, the shaded area of the graph highlights this gain. At any given frequency, the corresponding loop gain equals the vertical distance between the A and $1/\beta$ curves. The logarithmic scale of the graph makes this distance $\log(A) - \log(1/\beta) = \log(A\beta)$. Loop gain $A\beta$ represents the amplifier's reserve capacity to supply the feedback demand for gain. Where loop gain drops below unity, the closed-loop curve drops from the ideal A_{CLi}.

The A and $1/\beta$ curves graphically display this loop gain limit. Here, the $1/\beta$ curve represents the feedback demand. Loop gain meets this demand as long as the $1/\beta$ curve remains below the open-loop gain curve. However, at higher frequencies, the open-loop gain curve falls below the $1/\beta$ level. There, feedback demand exceeds the available amplifier gain and A_{CL} rolls off, following the amplifier open-loop response. This response roll off follows a –20 dB/decade slope for the single-pole response characteristic of typical op amps.

4

1/β INTERCEPT MARKS BANDWIDTH LIMIT

The bandwidth limit of most op amp circuits occurs at the 1/β intercept with the open-loop gain curve. Some circuits reduce bandwidth further, through reactive feedback elements, but all op amp circuits encounter a bandwidth limit at the 1/β intercept. Figure 6 illustrates this intercept and the coincident roll off of the A_{CL} response. By definition, the 3dB bandwidth limit occurs where A_{CL} drops from its DC value to 0.707 times that value. Analysis shows that this condition results at the intersection of the A and 1/β curves. These curves are actually magnitude responses and, at their intersection, their magnitudes are the same or $|A| = |1/β|$. Rearranging this result shows that the intercept occurs where the loop gain is $|Aβ| = 1$ at the frequency f_i. A phase shift of –90° accompanies this unity gain magnitude because of the single-pole roll off of gain A. Then, $Aβ = –j1$, at the intercept, and the denominator of the A_{CL} equation becomes $1 + 1/Aβ = 1 + j1$.

The $\sqrt{2}$ magnitude of this denominator drops circuit gain from A_{CLi} to $0.707A_{CLi}$. Thus, for frequency independent feedback factors, the 3 dB bandwidth occurs at the intercept frequency f_i. With frequency dependent feedback factors, the closed-loop response still rolls off following the intercept but this point may not be the 3dB bandwidth limit. Then, peaking or additional roll off in the closed-loop response curve moves the actual 3dB point away from f_i.

For the more common op amp applications, constant feedback factors permit a simple equation for the 3dB bandwidth. Single-pole responses characterize the open-loop roll offs of most op amps and virtually all 1/β intercepts occur in this single-pole range. There, the single-pole makes the gain magnitude simply $|A| = f_C/f$ where f_C is the unity-gain crossover frequency of the amplifier. Then, at the intercept, $f = f_i$ and $A = 1/β = f_C/f_i$. Solving for f_i defines the 3dB bandwidth for most op amp applications as

$$BW = f_i = βf_C$$

This result holds for all op amp applications having frequency independent β and single-pole op amp roll off.

Technically, the above bandwidth limit portrays only the small-signal performance of an op amp. In large-signal applications, slew rate limiting often sets a lesser bandwidth limit, especially in lower gain applications. There, the slew rate limit, S_r, imposes a power bandwidth limit of $BW_P = S_r/2πE_{op}$ where E_{op} is the peak value of the output voltage swing. This limit represents the only major performance characteristic of an op amp not directly related to the feedback factor β.

However, an indirect relationship still links large-signal bandwidth and β. The value of β helps determine which bandwidth limit, BW or BW_P above, applies in a given application. Both bandwidth limits set performance boundaries and the lower of the two prevails in large-signal applications. Higher values of β imply lower closed-loop gains and increase the frequency boundary set by $BW = βf_C$.

There, $BW_P = S_r/2πE_{op}$ generally produces the lower of the two boundaries, controlling the circuit bandwidth. Conversely, lower values of β reduce the $BW = βf_C$ boundary, making this the dominant limit. For a given application, compare the two limits to determine which applies.

BANDWIDTH ALSO RESTRICTS ERROR SIGNALS

The frequency dependence defined by the 1/β intercept also applies to the AC error sources of the previous Figure 2 analysis. That analysis showed that the input-referred errors of op amps transfer to the amplifier output through a gain of 1/β. However, 1/β does not include the high frequency limitations of the amplifier. Thus, the earlier analysis remains valid only for frequencies up to the 1/β intercept at f_i. Above this frequency, the amplifier lacks sufficient gain to amplify input error sources by a gain of 1/β. The bandwidth limit $BW = βf_C$ marks a response roll off that reduces amplification of signal and error alike. Beyond this BW limit, the gain available to error signals rolls off with the amplifier open-loop response. Here, the limited error signal magnitudes always invoke the small-signal, rather than slew-rate, bandwidth limit.

This error signal roll off produces the previously mentioned difference between 1/β and "noise gain". Beyond the intercept, the gain supplied to noise follows the amplifier response roll off even though the 1/β curve continues uninterrupted. For A_{CL}, the response roll off results from the denominator of this gain's equation. For error signal gain, adding this denominator to the original 1/β gain inserts the frequency dependance. This makes the closed-loop error gain

$$A_{CLe} = \frac{1/β}{1 + 1/Aβ}$$

Here, the added frequency dependance reduces the higher-frequency output errors calculated for the noise, CMRR and PSRR error sources.

For the noninverting case considered here, $A_{CLe} = A_{CL}$ but, for other cases, A_{CL} varies. Error gain A_{CLe}, however, remains the same. This gain always equals 1/β up to this curve's intercept with the amplifier open-loop response. Then, A_{CLe} rolls off with that response. Note that A_{CLe} above depends only upon the variables β and A. Any feedback model with β and A blocks configured like Figure 5 yields the same expression for A_{CLe}.

1/β INTERCEPT ALSO DEFINES STABILITY

The AC performance indications of the feedback factor also predict op amp frequency stability. The response plots that define bandwidth also communicate the phase shift of the feedback loop. Excess phase shift promotes response ringing or oscillation and the plot slopes indicate this phase shift directly. Mathematical analysis defines the stability indicators applied to the plots and an intuitive evaluation verifies these indicators.

Response plots like that of Figure 6 permit frequency stability evaluation directly from the curve slopes. Specifically, the slopes of the A and $1/\beta$ curves at the intercept indicate phase shift for a critical feedback condition. As mentioned, the intercept corresponds to a loop gain magnitude of $|A\beta| = 1$. If the loop phase shift reaches 180°, the loop gain at the intercept becomes $A\beta = -1$. Then, the denominator of A_{CL} equation reduces to $1 + 1/A\beta = 0$ making A_{CL} infinite. With infinite gain, a circuit supports an output signal in the absence of an input signal. In other words, the circuit oscillates and it does so at the intercept frequency f_i.

The relative slopes of the gain magnitude and $1/\beta$ curves reflect the phase shift of the feedback loop. The relationship between response slope and phase shift follows from the basic effects of response poles and zeros. A pole creates a –20dB/decade response slope and –90° of phase shift and a zero produces the same effects with opposite polarities. Additional poles and zeroes simply add response slope and phase shift in increments of the same magnitudes. The slope and phase correlation accurately predicts the loop phase shift when the critical intercept remains well separated from response break frequencies. Within a frequency decade of the intercept, any break frequency of the amplifier or feedback network requires the more detailed analysis described later. However, even in these cases, the response slopes provide insight into probable stability behavior.

Relying on the slope and phase correlation, the rate-of-closure guideline quickly approximates the phase shift of $A\beta$. Rate-of-closure is simply the difference between the slopes of the A and $1/\beta$ curves at the intercept. Both slopes communicate phase shift and the slope difference indicates the net phase shift of the loop. Figure 7 illustrates the slope and phase correspondence for two common feedback cases. There, two $1/\beta$ curves having different slopes intercept the gain magnitude curve $|A|$. The $1/\beta_1$ curve has the zero slope of resistive feedback networks and the rate-of-closure depends only upon the gain magnitude curve. This curve has the –20dB/decade slope common to most op amps. Together, the two curves develop a 20dB/decade slope difference, or rate-of-closure, for 90° of $A\beta$ phase shift.

In the feedback loop, the phase inversion of the op amp adds another 180° for a net phase shift of 270°. This leaves a phase margin of $\Phi_m = 90°$ from the 360° needed to support oscillation. For op amp stability analysis, the 180° phase shift from the amplifier phase inversion is automatic. Thus, op amp phase analysis simplifies, replacing the normal 360° stability criteria with a criteria of 180° of feedback phase shift. This convention applies in the examples that follow.

The second $1/\beta$ curve of Figure 7 illustrates the feedback condition of the basic differentiator circuit. This circuit produces a feedback demand curve represented in the figure by $1/\beta_2$. That curve slopes upward at +20dB/decade and intercepts the $|A|$ curve where the slope difference is 40dB/decade. Then, the rate-of-closure guideline indicates a feedback phase shift of 180°, leaving zero phase margin. This explains the inherent oscillation of the basic differentiator circuit.
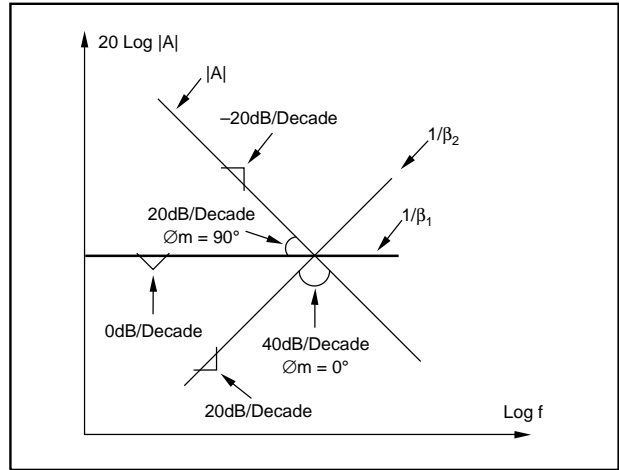

FIGURE 7. Plotted together, the $1/\beta$ and open-loop gain curves display a circuit's frequency stability conditions through the curves' rate-of-closure.

APPROXIMATION SIMPLIFIES PHASE ANALYSIS

As mentioned, the rate-of-closure criteria accurately predicts the $A\beta$ phase shift when no response break frequencies occur within a decade of the intercept. Other cases require more detailed phase analysis but this too simplifies with the Bode phase approximation.[5] This approximation produces a maximum error of 5.7°. Shown in Figure 8, this approximation predicts the phase effect of a response singularity through a straight line approximation. The actual phase shift introduced by the illustrated pole at f_p progresses through the arctangent curve shown. The actual phase shift at any frequency can be calculated from $\Phi = \text{Arctan}(f/f_p)$.

However, the Bode approximation provides quicker, visual feedback when examining response plots. This approximation simplifies the phase shift curve to a straight line having a slope of –45°/decade. This line centers on the frequency f_p, where the phase shift is 45°. From there, the approximation line predicts 0° at $0.1f_p$ and the full 90° at $10f_p$. Just these three reference points provide a quick visual indication of the effect a given response break produces at a frequency of
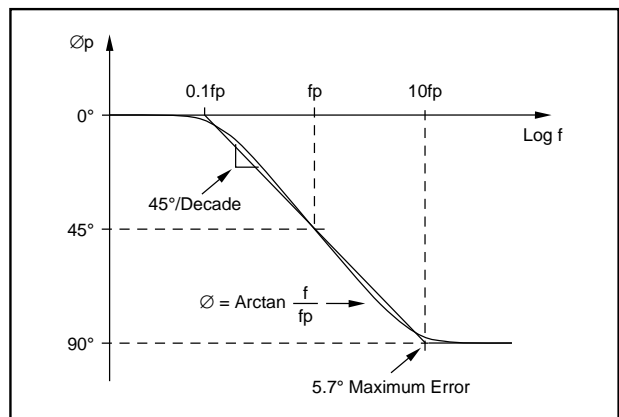

FIGURE 8. The Bode phase approximation refines phase analysis for cases where the rate-of-closure criteria looses accuracy.

interest. Outside the band of $0.1f_p$ to $10f_p$, a response break produces little influence. Near f_p, the response break introduces around 45° of phase shift.

In between these reference points, visual extrapolation approximates the phase shift. For example, consider a point midway between the f_p and $10f_p$ marks of the Log f scale. Note that this midpoint is a linear measure on the log scale. This requires no logarithmic conversion and visual perception of distance applies directly. At this midpoint, the phase approximation indicates a phase shift of approximately 45° + 0.5(45°) = 67.5°. Similarly, at a point two-tenths of the way between $0.1f_p$ and f_p the approximation indicates 0.2 (45°) = 9°. These analyses require no knowledge of the actual frequencies represented by the example points. In contrast, exact analysis with the arctangent relationship first requires conversion of the linear distance observed into the equivalent frequency of the log f scale. Then, the arctangent relationship must be calculated.

Figure 9 illustrates the application of the Bode phase approximation to the stability indication of the $1/\beta$ intercept. In the figure, the intercept occurs where the open-loop gain response has a slope of –20dB/decade. The rate-of-closure guideline suggests 90° of loop phase shift. However, a second amplifier pole at f_p develops a –40dB/decade slope in the open-loop gain response. As shown, the pole at f_p occurs less than a decade from the intercept. This limited separation compromises the simple rate-of-closure indication.
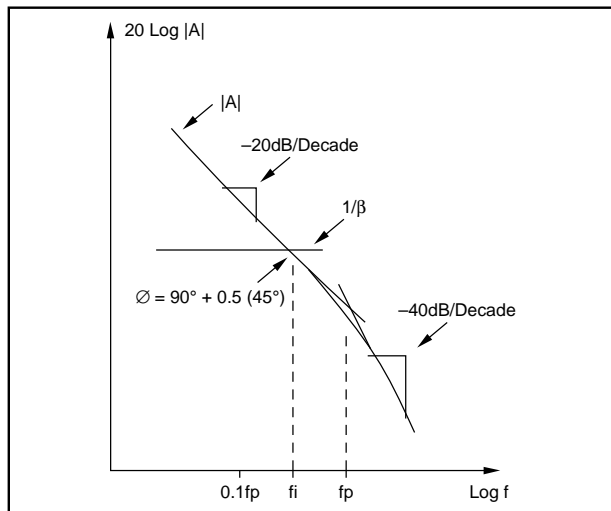

FIGURE 9. Application of the Bode approximation defines the phase effects of response breaks that occur less than a decade from the intercept at f_i.

Then, the Bode phase approximation estimates the phase effect of f_p at the intercept f_i. As shown, f_i occurs below f_p so the effect is less than 45°. Refinement of this initial estimate follows from the linear distance separating f_p and f_i on the plot. This linear distance represents a fraction of a frequency decade. The fraction equals this distance divided by the linear distance between f_p and $0.1f_p$. Visual reading of the

example shown places f_i about midway between f_p and $0.1f_p$. This communicates a phase effect from f_p of 0.5(45°) = 22.5° at the intercept frequency f_i. Adding this to the 90° produced by the –20db/decade gain slope results in a net loop phase shift of 112.5°. This leaves 67.5° of phase margin from the 180° of feedback phase shift required for oscillation.

INTUITIVE ANALYSIS EXPLAINS OSCILLATION

With op amps, conventional insight into the cause of amplifier oscillation can be misleading. In the general amplifier case, high gain combined with high phase shift promotes oscillation. In the op amp case, these conditions often exist together without producing instability. The distinction lies in the simultaneous gain and phase conditions required for op amp oscillation. At lower frequencies, high loop gain prevents oscillation by attenuating the amplifier's input error signal. At higher frequencies, lack of loop gain restricts the output signal to similarly prevent oscillation. In between, the loop gain reaches a point where the high and low frequency limitations cross, satisfying the gain condition for oscillation. Still, the feedback phase shift at this crossover must reach 180° to produce oscillation.

To illustrate this gain and phase combination, Figure 10 demonstrates the basic requirements for op amp oscillation. This figure grounds the normal signal input of the circuit to remove the effect of any applied signal upon the output voltage. With the grounded input, only the gain error signal, $-e_o/A$, excites the input circuit. This signal must independently produce the output signal in order to sustain an oscillation. The circuit amplifies the gain error signal by the closed-loop gain A_{CL}, producing $e_o = A_{CL}(-e_o/A)$. In turn, this output signal reflects back through the amplifier, producing the attenuated input signal $-e_o/A$. If this circuit gain and attenuation cycle supports an output signal, it is self-sustaining oscillation.
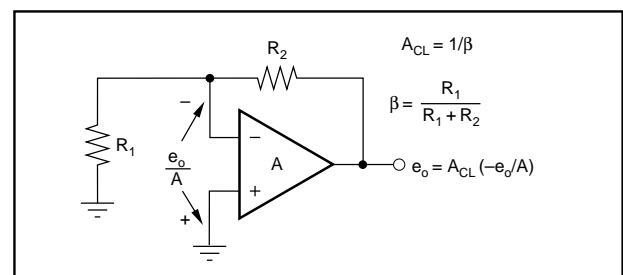

FIGURE 10. To sustain oscillation, error signal e_o/A and gain A_{CL} must support the output voltage in the absence of an applied input signal.

At lower frequencies, $A_{CL} = 1/\beta$, making the oscillation condition $e_o = -e_o/A\beta$. To sustain oscillation, the circuit must satisfy this equality and only two solutions do, $e_o = 0$ and $A\beta = -1$. The $e_o = 0$ solution indicates an oscillation of zero amplitude, representing the stable state. The $A\beta = -1$ solution represents the actual oscillation state, as noted in the previous mathematical analysis. This second solution places very specific magnitude and phase requirements upon the

loop gain $A\beta$. The condition $A\beta = -1$ requires that $|A\beta| = 1$ in combination with 180° of phase shift for the minus sign.

Consider the magnitude requirement first. If $|A\beta|$ is too large, the circuit conditions would require $|e_o| > |e_o/A\beta|$. This condition can not be self sustaining. Here, the attenuated input error signal e_o/A, when amplified by a gain of $1/\beta$, remains too small to support the required e_o. Only when the attenuating gain, A, equals the amplifying gain, $1/\beta$, does the circuit meet the magnitude condition for oscillation. Expressing this in an equation, $|A| = |1/\beta|$, repeats the previous mathematically derived condition for oscillation. Only at the intercept of the A and $1/\beta$ curves do their magnitudes become equal. Only then does the circuit fill the magnitude condition for oscillation.

At this intercept, oscillation also requires 180° of feedback phase shift. If $A\beta$ lacks 180° of phase shift, then the minus sign of the $A\beta = -1$ condition remains unsatisfied, preventing oscillation. Further, oscillation only results when this phase condition coincides with the magnitude condition above. An $A\beta$ phase shift of 180° at frequencies other than the intercept frequency does not produce oscillation. At those other frequencies, the circuit fails to meet the magnitude condition for oscillation.

Composite amplifiers permit a graphical illustration of this combined oscillation requirement. These amplifiers inherently produce the 180° phase shift required for the minus sign. They consist of two op amps connected in series and each amplifier contributes a –20dB/decade slope to the composite open-loop gain. This produces a –40dB/decade gain slope as illustrated by Figure 11. This slope indicates 180° of phase shift over most of the amplifier's useful frequency range. Thus, composite amplifiers meet the phase condition for oscillation over a broad range.
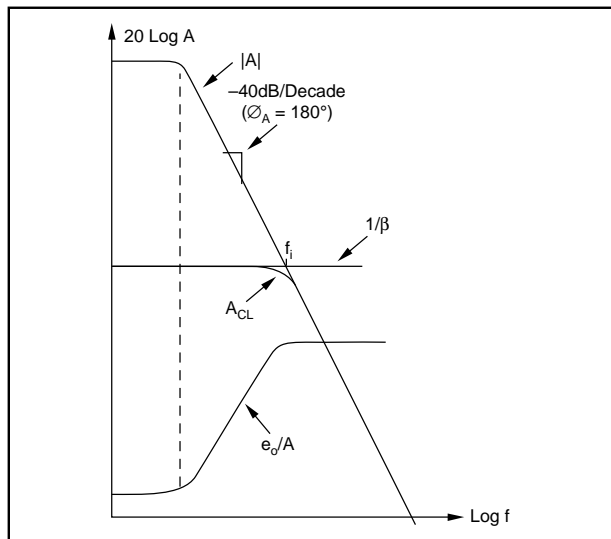

FIGURE 11. A composite amplifier response illustrates the fact that 180° of phase shift fails to support oscillation where e_o/A lacks sufficient magnitude.

Over the same frequency range, the composite amplifier provides high open-loop gain. This gain and phase combination might first suggest stability problems over the entire range. However, the high open-loop gain actually serves to stabilize the circuit through the circuit's loop gain. High values for A increase the loop gain $A\beta$ to prevent the magnitude equality $|e_o| = |e_o/A\beta|$ required for oscillation. It does so by limiting the e_o/A error signal as illustrated in the figure. At lower frequencies, high levels of open-loop gain A reduce this input signal to a level insufficient to support oscillation.

The e_o/A curve rises as gain A declines but flattens when A_{CL} declines. The rise in e_o/A must reach a certain level to support the oscillation condition of $e_o = -A_{CL}(e_o/A)$. Also, to support this condition, the high-frequency roll off of A_{CL} must not excessively reduce this gain. Otherwise, the amplification of e_o/A by A_{CL} fails to develop sufficient e_o to sustain oscillation. Only one point in the plots satisfies this oscillation condition. As described before, where $A = 1/\beta$, e_o/A reaches the level required to support oscillation. This intercept also marks the peak value for $A_{CL}(e_o/A)$. Beyond there, A_{CL} rolls off with gain A, reducing e_o and leveling the e_o/A curve. With a level e_o/A curve, the A_{CL} roll off also rolls off the quantity $A_{CL}(e_o/A)$.

Before this intercept, e_o/A remains too small to support oscillation. After the intercept, the amplifier lacks the A_{CL} needed to sustain oscillation. Thus, before or after the intercept, 180° of feedback phase shift does not compromise stability. This phase shift produces oscillation only if present at the frequency of the intercept. There, gain magnitude conditions always permit oscillation given the required 180° phase condition. Phase compensation reduces this phase shift for the composite amplifier.

The $1/\beta$ intercept represents a critical mass point for frequency stability. There, the magnitude of the gain error and the feedback phase shift must both reach specific levels to support oscillation. Despite the very specific requirements for oscillation, the greatly varied applications of op amps make this critical mass condition all to easy to find. To contend with this, the $1/\beta$ curve presents visual prediction of the problem and provides insight into its solution.

REFERENCES

1. Graeme, J. G., *Feedback Models Reduce Op Amp Circuits to Voltage Dividers*, EDN, June 20, 1991, pg 139.

2. Graeme, J. G., *Op Amp Distortion Measurement Bypasses Test Equipment Limits*, EDN, February 17, 1992, pg 133.

3. Black, H., *Stabilized Feedback Amplifiers*, Bell System Technical Journal 13, 1/35.

4. Graeme, J. G., *Feedback Plots Offer Insight Into Operational Amplifiers*, EDN, January 19, 1989, pg 131.

5. Tobey, G. E., Graeme, J. G., Huelsman, L. P., *Operational Amplifiers; Design and Applications*, McGraw-Hill, 1971.